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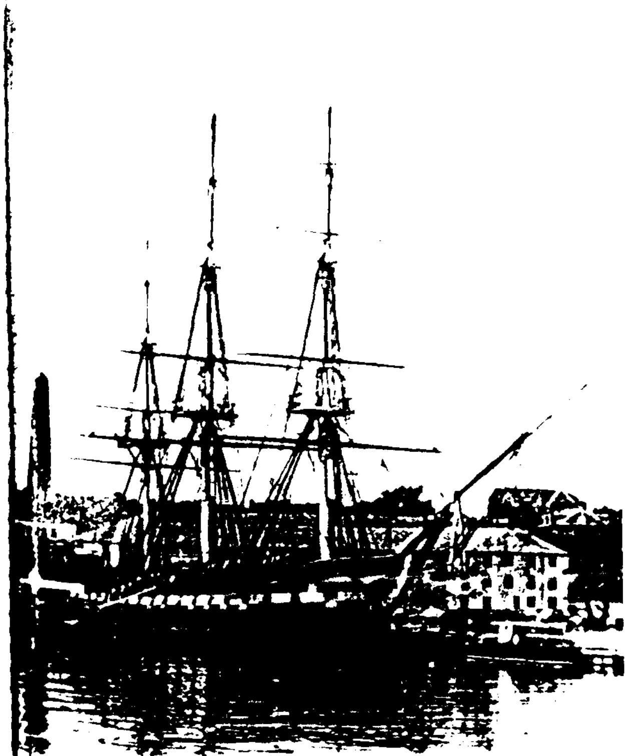

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USE OF LITERAL INFORMATION IN MULTI-TARGET DATA ASSOCIATION

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Abstract

It has been shown that literal information can enhance geolocation information in the multi-target tracking and data association problem. This paper continues previous efforts in establishing a systematic approach to the combination of both types of information using membership functions based upon multiple-valued logic. Filters are established for literal and non-numerical attributes, somewhat analogous to the well-known Kalman filter. The major result, however, is an improvement and clarification of a previous theorem establishing asymptotic forms for the posterior possibility distribution of the unknown data association parameter as information granularity decreases and as inference rule structures become more definitive.

1. Introduction

The multi-target tracking and data association (or as commonly called, "correlation") problem still remains the center of much activity and interest. In the past, emphasis was placed upon the use of only geolocation data-i.e., information containing reports on (usually) two- or three-dimensional target positions, together with possible velocities, accelerations and related equations of motion parameters, [1-3]. More recently, an effort has been carried out in utilizing in a more rigorous or systematic manner other types of information, including various sorts of sensor system parameters that could also be formally treated in terms of equations of motion. In addition, use of attribute information of radically different natures is also sought. This includes visual sightings, classifications, and other, often non-numerical and linguistic-based (or narrative), information which cannot be treated from traditional statistical analysis. This work is presented in [4-6], based upon and related to, in general, previous and ongoing research in possibility and fuzzy set theory and its realtionships to classical probability theory and multi-valued logic and set theory [7-9].

In this paper we continue to attempt to establish a unified approach to the integration of both types of information for the multi-target tracking and data association problem. However, since the results are carried out on a general level, with suitable modifications, applications to medical diagnosis, fault determination, and other problems involving non-sequential knowledge-based systems can be established [9].

As stated before, this work still unfortunately requires both a sense of "art" as well as science in achieving its goal.

2. Basic Problem

The basic problem can be stated as follows:

Observed or reported data arrives, usually indexed by time t_m, attribute type k, and sensor source u. For purposes of simplicity, we will omit the last index and assume the multiple sensor source problem is resolved and plays no role in the analysis here. Attributes A_k, k=1,2,...,M", can be either objective or statistical in nature or they can be subjective or linguistic in nature. Denoting the natural domain of values (which can be numbers, vectors of numbers, linguistic labels, etc.) of each attribute A_k as $D_k \triangleq \mathrm{dom}(A_k)$, statistical attributes typically have their domains $D_k \subseteq \mathbb{R}^{n_k}$ with error distributions in the form of classical probability functions, typically being discretized versions of Gaussian or mixtures of Gaussian distributions. Thus, in general, unless representing dirac or mass-point distributions, the error probability functions are not normable, i.e., they have maximal values less than unity (due to the obvious constraint of probabilities adding up to one). On the other hand, subjective attributes A_k typically have D_k being some finite set of labels with no natural spacial ordering present, unless D_k represents a set of n_k-tuples of measurement quantities, where for example, A_7= "large" could have D_7 consisting of pairs (a,b), with a being weight in lbs and b length in meters. The error distributions corresponding to subjective attributes are often in the form of normable possibility functions - but not probobility functions - obtained from a panel of experts, in contrast to the geolocation-physical derivation of statistical attribute error distributions. For motivation for such subjective distributions, representing overlapping and/or vague compound events - in contradistinction to statistical distributions representing disjoint exhaustive events- see again [4]. Each error distribution represents the conditional possibility (possibilities include probabilities as special cases) that a particular attribute domain value is actually present, given an observed/reported domain value. Whether statistical or subjective, it is assumed that at least theoretically each such distribution is obtainable.

In addition to error distributions and observed data, a collection of inference rules is assumed to be present. Each such rule R_v consists of an antecedent part ant_v and a consequent part $conseq_v$ connected by an implication operation. ant_v consists of the conjunction (more generally, some combination of conjunctions, disjunctions, and/or negations) only of modified matching tables for each attribute from some subset of all the attributes. Each modification $mod_{v,k}$ of basic matching table M_k for A_k is assumed to be in the form of an intensification or strengthening or an extensification or weakening. More generally, modifications can involve positive and/or negative forms. For simplicity, we assume throughout here that only positive forms are present. (See [10] for more background on modifiers and hedges for attributes.) In ordinary English, intensifications can be represented in a simple order as e.g. ..."slightly very",..,"very",...,"very,very",..,"extremely",... Extensifications can be expressed as e.g. ..."very very little of",..,"very little of",..,"more or less of",..,"moderate amount of",.. The consequence consists of a single modification mod_v (again, either a positive intensification or extensification) of the basic data association level.

Then, as data arrives, track histories are built up (with some eventually discarded) sequentially in time, consisting of those data reports which are adjudged as belonging to the same target source. The decision on associating together or not a given pair - a previously established or tentative track history with a new report - is based upon the application of the set of inference rules and error distributions upon the two data vectors, where it is understood that the previously established track history i is suitably updated for comparisons with the new report j. Of course, for a given track history the updating of the data history for a statistical attribute, under linear-Markov or Gaussian assumptions, can be made using the now well-established Kalman filter [11]. On the other hand, the very different structured subjective attributes require another approach. For example, A_{10} might be "color of flag" with D_k = {red, red-orange, orange,..., blue,..., black striped,..}, or A_{17} might be "class" with $D_k = \{C_1, C_2, \ldots\}$. How do we "filter" or predict among such values? Thus an attribute filter is sought for these cases. But after a moment's thought, the answer to this problem is relatively simple: obtain the possibility analogues for the probability function situation. This is spelled out in section 4. (For a schematic view of one cycle of the data association process from one sampling time to the next, see Figure 1 at the end of section 4.)

3. Definitions & Notation

The notation used here is somewhat different from the previous and related paper [6], due to the ongoing effort to employ the simplest, yet most accurate notation.

$$\underline{\theta}^{(i,j)} \stackrel{d}{=} (\theta_v^{(i,j)})_{v=1,\ldots,N} \quad ,$$

$\theta_v^{(i,j)}, \theta \in \mathrm{dom}(C\delta^{(i,j)}) = [0,1]$ is unknown parameter representing data association level between track histories i and j ; $C\delta^{(i,j)}$ is the attribute representing "correlation" between i and j.

$A_1, A_2, \ldots, A_{M'}$ is the set of attributes which are statistical ;

$A_{M'+1}, A_{M'+2}, \ldots, A_{M''}$ is the set of subjective attributes.

To indicate dependency and change due to sampling time t_m , where

$$t_0 < t_1 < t_2 < t_3 < \cdots$$

and track history i , denote A_k as $A_{k,m}^{(i)}$ with

$$\mathrm{dom}(A_{k,m}^{(i)}) \stackrel{d}{=} D_{k,m}^{(i)}$$

similarly replacing D_k;

$$D_m^{(i)} \stackrel{d}{=} \mathop{\times}_{k=1}^{M''} (D_{k,m}^{(i)}) \; .$$

$Z_{k,m}^{(i)}$, $\mathring{Z}_{k,m}^{(i)} \in D_{k,m}^{(i)}$ are the possible true , updated/smoothed-observed values for $A_{k,m}^{(i)}$, respectively.

$$\mathring{Z}_{k,m}^{(i,j)} \stackrel{d}{=} (\mathring{Z}_{k,m}^{(i)}, \mathring{Z}_{k,m}^{(j)}) \; ,$$

$$\mathring{Z}_k^{(i,m_0)} \stackrel{d}{=} (\mathring{Z}_{k,m}^{(i)})_{m \le m_0} \quad , \text{ for any } m_0 = 0,1,2,\ldots$$

$$\mathring{Z}^{(i,m_0)} \stackrel{d}{=} (\mathring{Z}_m^{(i)})_{m \le m_0} \; ,$$

$$\mathring{Z}_m^{(i)} \stackrel{d}{=} (\mathring{Z}_{k,m}^{(i)})_{k=1,\ldots,M''} \quad , \text{ etc.}$$

Similar notation holds for the true possible attribute values.

We use prime(') notation to indicate the statistical part and double prime notation(") to indicate the subjective part:

$$Z_m^{(i)} = (Z_m'^{(i)}, Z_m''^{(i)})$$

$$Z_m'^{(i)} \stackrel{d}{=} (Z_{k,m}^{(i)})_{k=1,\ldots,M'}$$

$$Z_m''^{(i)} \stackrel{d}{=} (Z_{k,m}^{(i)})_{k=M'+1,\ldots,M''}$$

$$D_m^{(i)} = D_m'^{(i)} \times D_m''^{(i)} \quad , \text{ etc.}$$

We use ϕ to indicate possibility function; $\phi(\cdot|\cdot\cdot)$ to indicate conditional or dependent possibility function.

$P_{k,m}^{(i)}$ is the index denoting the conditional error possibility function for $A_{k,m}^{(i)}$. Thus

$$\phi(\cdot|\cdot\cdot; P_{k,m}^{(i)}) : D_{k,m}^{(i)} \times D_{k,m}^{(i)} \to [0,1]$$

and

$$\phi(Z_{k,m}^{(i)} | \mathring{Z}_{k,m}^{(i)}; P_{k,m}^{(i)})$$

$= \text{possibility } Z_{k,m}^{(i)}$ is true value for $A_{k,m}^{(i)}$ given $\mathring{Z}_{k,m}^{(i)}$ is updated observed value.

Also,

$$P_k^{(i,m_0)} \stackrel{d}{=} (P_{k,m}^{(i)})_{m \le m_0} = (P_k'^{(i,m_0)}, P_k''^{(i,m_0)})$$

$$P_m^{(i)} = (P_m'^{(i)}, P_m''^{(i)}) = (P_{k,m}^{(i)})_{k=1,\ldots,M''} \quad , \text{ etc.}$$

Four special possibility functions are indicated by:

ϕ_{nt} :$[0,1] \rightarrow [0,1]$,a decreasing or at least non-increasing function with boundary conditions coinciding with usual two-valued logic:

$$1=\phi_{nt}(0) \ ; \ 0=\phi_{nt}(1) ,$$

representing negation or general set or attribute complement ;

$\phi_{\&}$: $[0,1]^2 \rightarrow [0,1]$, a nondecreasing function in each of its arguments, which is continuous,$\leq$ min, associative, symmetric, and possesses the boundary conditions reducing to the two-valued logic ones:

$$0=\phi_{\&}(x,0)=\phi_{\&}(0,x) \ ; \ x=\phi_{\&}(x,1)=\phi_{\&}(1,x),$$

for all $x \in [0,1]$. This operator may be unambiguously extended to an arbitrary finite number of arguments and represents conjunction or general set or attribute intersection;

ϕ_{or}: $[0,1]^2 \rightarrow [0,1]$, a nondecreasing function in each of its arguments, which is continuous,$\geq$ max, associative, symmetric, and possesses the boundary conditions reducing to the two-valued logic ones:

$$1=\phi_{or}(x,1)=\phi_{or}(1,x) \ ; \ x=\phi_{or}(x,0)=\phi_{or}(0,x),$$

for all $x \in [0,1]$. This operator may be unambiguously extended to an arbitrary finite number of arguments and represents disjunction or general set or attribute union;

$\phi_{\rightarrow}$:$[0,1]^2 \rightarrow [0,1]$, a non-increasing function in its first argument and a nondecreasing function in its second argument with the two-valued logic boundary conditions

$$1=\phi_{\rightarrow}(0,x) \ = \phi_{\rightarrow}(x,1) \ ; \ 0 = \phi_{\rightarrow}(1,0),$$

for all $x \in [0,1]$. This operator represents logical implication : "if(·) then (··)".

$\phi_{\&}$ is also called in the literature a t-norm and ϕ_{or} is called a t-conorm. See [12] or [9], Chapter 2.3.6 for more details on these function classes.

Next,let R_v denote the v^{th} inference rule, with associated possibility function

$$\phi(\cdot|\cdot\cdot;R_v):[0,1] \times D_m^{(i,j)} \rightarrow [0,1],$$

somewhat abusing notation, where there is an associated index set $J_v \subseteq \{1,2,\ldots,M''\}$, where as usual,

$$J_v' \triangleq J_v \cap \{1,\ldots,M'\}, \ J_v'' \triangleq J_v \cap \{M'+1,\ldots,M''\} ,$$

so that

$$\phi(\theta_v|Z_m^{(i,j)};R_v) \triangleq \phi_{\rightarrow}(\phi_{ant_v}(Z_m^{(i,j)}),\phi_{conseq_v}(\theta_v)),$$

$$\phi_{ant_v}(Z_m^{(i,j)})= \underset{k\in J_v}{\phi_{\&_1}}(\phi_{mod_{k,v}}(\phi_{M_k}(Z_{k,m}^{(i,j)}))),$$

$$\phi_{conseq_v}(\theta_v) = \phi_{mod}(\phi_{Co}(\theta_v)) .$$

Denote also the index set

$$R \triangleq (R_v)_{v=1,\ldots,N} .$$

Then we write for the overall error distribution:

$$\phi(Z_m^{(i,j)}|\mathring{Z}_m^{(i,j)},P_m^{(i,j)})$$
$$\triangleq \ \phi_{\&_2}(\phi(Z_m'^{(i,j)}|\mathring{Z}_m'(i,j);P_m'(i,j)),\phi(Z_m''(i,j)|\mathring{Z}_m''(i,j);P_m''(i,j))),$$

$$\phi(Z_m'^{(i,j)}|\mathring{Z}_m'(ij);P_m^{(i,j)})$$
$$\triangleq \underset{(k=1,\ldots,M')}{\phi_{\&'}}(\ \phi(Z_{k,m}^{(i,j)}|\mathring{Z}_{k,m}^{(i,j)};P_{k,m}^{(i,j)}));$$

$$\phi(Z_{k,m}^{(i,j)}|\mathring{Z}_{k,m}^{(i,j)};P_{k,m}^{(i,j)})$$
$$\triangleq \ \phi_{\&_3}(\phi(Z_{k,m}^{(i)}|\mathring{Z}_{k,m}^{(i)},P_{k,m}^{(i)}),\phi(Z_{k,m}^{(j)}|\mathring{Z}_{k,m}^{(j)};P_{k,m}^{(j)})), \qquad (1)$$

with $\phi(Z_m''(i,j)|\mathring{Z}_m''(i,j),P_m''(i,j))$ defined similarly in terms of $\phi_{\&''}$.

For the overall inference rule effect, we have:

$$\phi(\underline{\theta}|Z_m^{(i,j)};R) \triangleq \underset{(v=1,\ldots,N)}{\phi_{\&_4}} \ (\phi(\theta_v|Z_m^{(i,j)};R_v)). \qquad (2)$$

Other notation and definitions will be introduced as needed. From now on, only the most important changes in t-norms and t-conorms will be specially indicated; otherwise, the generic notation $\phi_{\&}$, ϕ_{or} will be used.

4. Basic Analysis

The following simple theorem is proven easily using the basic properties of conditional possibility functions as given in [6],[7], or [9] and can serve as the desired attribute filter. Note also that in general the distinction(omitting $P_k^{(i,m)}$)

$$\phi(\mathring{Z}_{k,m}^{(i)}|\mathring{Z}_k^{(i,m-1)}) \neq \phi(Z_{k,m}|\mathring{Z}_k^{(i,m-1)}) \ ,$$
$$\text{for } Z_{k,m}^{(i)}=\mathring{Z}_{k,m}^{(i)} .$$

Theorem 1

Suppose (omitting the $P^{(i)}$ and $($ $^{(i)}$) for any k, k=M'+1,...,M",

$$\phi(\mathring{Z}_{k,m}|Z_{k,m},\mathring{Z}_k^{(m-1)})=\phi(\mathring{Z}_{k,m}|Z_{k,m})$$

$$\phi(Z_{k,m}|Z_{k,m-1},\mathring{Z}_k^{(m-1)})=\phi(Z_{k,m}|Z_{k,m-1})$$

are known functions with $\phi(Z_{k,m},Z_{k,m-1}|\mathring{Z}_k^{(m-1)})$,etc. normable with respect $\phi(Z_{k,m}|\mathring{Z}_k^{(m-1)})$ and without loss of generality suppose that $\phi(Z_{k,m}|\mathring{Z}_k^{(m)})$ is normable. Then:

(i) $\phi(\mathring{Z}_{k,m}|\mathring{Z}_k^{(m-1)})=\underset{Z_{k,m}}{\max}\left(\phi_{\&}(\phi(Z_{k,m}|\mathring{Z}_k^{(m-1)}),\phi(\mathring{Z}_{k,m}|Z_{k,m}))\right)$

(ii) $\phi(Z_{k,m}|\mathring{Z}_k^{(m-1)})=\underset{Z_{k,m-1}}{\max}\left(\phi_{\&}(\phi(Z_{k,m-1}|\mathring{Z}_k^{(m-1)}),\phi(Z_{k,m}|Z_{k,m-1}))\right)$

(iii) $\phi(Z_{k,m}|\mathring{Z}_k^{(m)})$ is implicitly obtainable from

$$\phi_{\&}(\phi(Z_{k,m}|\mathring{Z}_k^{(m)}),\phi(\mathring{Z}_{k,m}|\mathring{Z}_k^{(m-1)}))$$
$$= \phi_{\&}(\phi(Z_{k,m}|\mathring{Z}_k^{(m-1)}),\phi(\mathring{Z}_{k,m}|Z_{k,m})).$$

Next, for purpose of completeness, the full PACT algorithm [4],[6],[9] (Chapter 9) will be presented. For justification of the results see the above references.

The final posterior possibility function is $\phi(\theta|diag,\mathring{Z}_m^{(i,j)};R,P_m^{(i,j)})$ obtainable implicitly from the equation, again using simple notation,

$$\phi_{diag}(\theta|\mathring{Z}_m^{(i,j)};R,P)=\phi_{\&}\Big(\phi(\theta|diag,\mathring{Z}_m^{(i,j)};R,P),\ \phi_{univ}(diag)\Big), \quad (3)$$

$$\phi_{univ}(diag)=\max_{(0\le\theta\le 1)}(\phi_{diag}(\theta|\mathring{Z}_m^{(ij)};R,P)), \quad (4)$$

$$\phi_{diag}(\theta|\mathring{Z}_m^{(ij)};R,P)\stackrel{d}{=}(\phi(\underline{\theta}|\mathring{Z}_m^{(ij)};R,P))_{\theta_v=\theta,\ v=1,\dots,N}, \quad (5)$$

$$\phi(\underline{\theta}|\mathring{Z}_m^{(i,j)};R,P)=\phi_{or}{}_{(Z_m(ij)\in D_m^{(ij)})}(\phi(\theta,Z_m^{(ij)}|\mathring{Z}_m^{(ij)};R,P)), \quad (6)$$

$$\phi(\underline{\theta},Z_m^{(ij)}|\mathring{Z}_m^{(ij)};R,P)=\phi_{\&}(\phi(\underline{\theta}|Z_m^{(ij)},\mathring{Z}_m^{(ij)};R,P),\ \phi(Z_m^{(ij)}|\mathring{Z}_m^{(ij)};R,P)), \quad (7)$$

where

$$\phi(\underline{\theta}|Z_m^{(ij)},\mathring{Z}_m^{(ij)};R,P)=\phi(\underline{\theta}|Z_m^{(ij)};R), \quad (8)$$

$$\phi(Z_m^{(ij)}|\mathring{Z}_m^{(ij)};R,P)=\phi(Z_m^{(ij)}|\mathring{Z}_m^{(ij)};P) \quad (9)$$

are obtainable as in section 3.

Furthermore, in general, ϕ_{or} above breaks up into statistical and subjective components:

$$\phi(\underline{\theta}|\mathring{Z}_m^{(ij)};R,P)=\phi_{or''}{}_{(Z''_m(ij)\in D''_m(ij))}(\phi_{or'}{}_{(Z'_m(ij)\in D'_m(ij))}(G(\underline{\theta},Z_m^{(ij)};R,P))) \quad (10)$$

where

$$G(\underline{\theta},Z_m^{(ij)};R,P)\stackrel{d}{=}\phi_{\&}(\phi_H(\underline{\theta},Z_m^{(ij)};\mathring{Z}''_m(ij);R,P''),\ \phi(Z'_m(ij)|\mathring{Z}'_m(ij);P')), \quad (11)$$

$$\phi_H(\underline{\theta},Z_m^{(ij)};\mathring{Z}''_m(ij);R,P'')\stackrel{d}{=}\phi_{\&}(\phi(\underline{\theta}|Z_m^{(ij)};R),\ \phi(Z''_m(ij)|\mathring{Z}''_m(ij);P'')), \quad (12)$$

where $\phi_{or''}$ may be chosen as max.

Since in general $\phi_{Co}(\theta)$ is a nondecreasing function in θ with value 1 at $\theta=1$, and if all modifiers are chosen in the positive sense as discussed in section 2 so that each $\phi_{mod_{k,v}}(x)$ and $\phi_{mod_v}(x)$ are nondecreasing functions of x with unity values at $x=1$, such as is the case with exponentials, then it follows from the property of $\phi_{\&}$ that the final posterior possibility function is formally like a distribution function except for not necessarily being 0 at $\theta=0$. Hence a reasonable measure of central tendency for θ so described is, using formally expectation notation,

$$E(\theta|diag,\mathring{Z}_m^{(ij)};R,P)\stackrel{d}{=}\int_{\theta=0}^{1}\theta\cdot d\phi(\theta|diag,\mathring{Z}_m^{(ij)};R,P)$$
$$=1-\int_{\theta=0}^{1}\phi(\theta|diag,\mathring{Z}_m^{(ij)};R,P)\cdot d\theta. \quad (13)$$

(See also [6] for further discussion.)

A simplified outline of a data association procedure utilizing the PACT algorithm as described above is given in Figure 1.

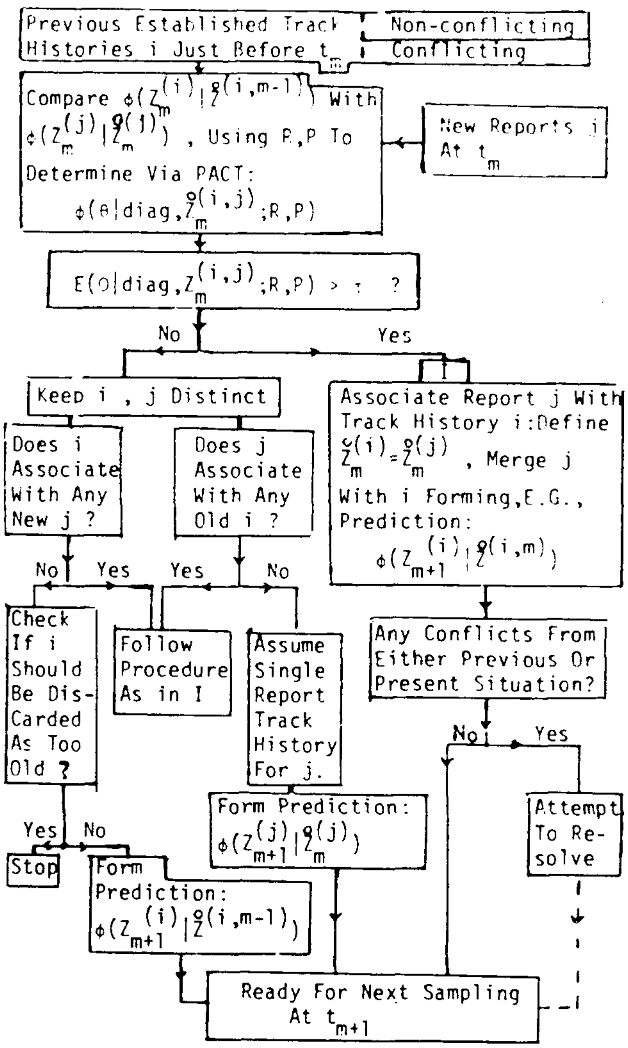

Figure 1. Flow Chart for Data Association From One Data Sampling Time to the Next

5. Some Asymptotic Results

Given the above scheme for obtaining the final posterior possibility function for the level of data association between two track histories, the following natural question arises: Can some analogue with the classical case of statistical consistency be established here? In [6] it was shown that as the fineness of the domains of statistical attributes increased- and dually, the granularity decreased- under certain reasonable conditions, the posterior function for data association converged to a computable function of a statistical expectation. In turn, the behavior of this limiting expression as inference rules and error distributions became more precise was considered, leading to, under perhaps too stringent conditions, a form of consistency. In order to clarify this further, a similar result, under weakened conditions and with

more specific applications, will be presented.

Suppose the following additional assumptions hold relative to the basic situation used to determine the posterior function of θ:

(a) For all $y \in \mathbb{R}^{nk}$, $f^{(\ell)}_{k,m}(\cdot|y): \mathbb{R}^{nk} \to \mathbb{R}^+$ is a bounded continuous conditional probability density function, for $k=1,\ldots,M'$, $\ell=i,j$, for some chosen track histories i,j.

(b) For each integer p>1, let $D^{(\ell)}_{k,m,p}$ be the p^th discretization and truncation of $D^{(\ell)}_{k,m}$ with $\Delta^{(\ell)}_{k,m,p}$ denoting the mesh of $D^{(\ell)}_{k,m,p}$ (i.e., maximal length of any rectangle formed within the truncation area), so that, in any sense, $\lim_{p\to+\infty} D^{(\ell)}_{k,m,p} = \mathbb{R}^{nk}$ and $\lim_{p\to+\infty} \Delta^{(\ell)}_{k,m,p} = 0$; in addition, replace each $\phi(Z^{(\ell)}_{k,m}|\hat{Z}^{(\ell)}_{k,m};P^{(\ell)}_{k,m})$ by the approximating probability function

$$\phi_p(Z^{(\ell)}_{k,m}|\hat{Z}^{(\ell)}_{k,m};P^{(\ell)}_{k,m}) = f^{(\ell)}_{k,m}(Z^{(\ell)}_{k,m}|\hat{Z}^{(\ell)}_{k,m})\Delta^{(\ell)}_{kmp} \quad (14)$$

and denote accordingly all computations involving this replacement, for $k=1,\ldots,M'$, $\ell=i,j$, with p.

(c) $\phi_{\&'}$ $(\phi_{\&_3}(x_{k,\ell}))$ is analytic about $x_{k,\ell}=0$, $k=1,\ldots,M'$, $\ell=i,j$.

(d) $\phi_{or'}$ is an Archimedean t-conorm with generator function h: $[0,1] \to \mathbb{R}^+\cup\{+\infty\}$ -which is continuous, non-increasing, with h(1)=0 and $h(0) \le +\infty$ and is such that

$$\phi_{or'}(x_1,\ldots,x_q) = 1-h^{-1}(\min(\sum_{\omega=1}^{\bar{q}} h(1-x_\omega),h(0))), \quad (15)$$

for all $x_\omega \in [0,1]$, q=1,2,... . (See, e.g., [12] and eq.(23).)

(e) Referring to (d), $h(1-\phi_\&(x,y))$ is analytic about x=y=0 in $[0,1]^2$.

(f) $\phi(\underline{\theta}|Z^{(i,j)}_m;R)$ is a continuous function in $Z'^{(i,j)}_m$ allowed to be arbitrary in $\bigtimes_{k=1}^{M'}(\mathbb{R}^{nk}\times\mathbb{R}^{nk})$. (This is guaranteed, if ϕ_+ is continuous in its first argument and ϕ_{mod} is continuous over [0,1] for $k=1,\ldots,M'$, $v=1,\ldots,N_k^v$)

Next, define

$$\kappa \overset{d}{=} \left(\partial^{2M'}(\underset{k=1,\ldots,M'}{\phi_{\&'}}\ (\underset{\ell=i,j}{\phi_{\&_3}}(x_{k,\ell})))/\partial x_{1,i}\partial x_{1,j}\cdots\partial x_{M',j}\right)_{\substack{x_{k,\ell}=0,\\ k=1,\ldots,M',\\ \ell=i,j}} \quad (16)$$

and

$$\nu(x) \overset{d}{=} -(d\,h(s)/ds)_{s=1}\cdot(\partial\phi_\&(x,y)/\partial y)_{y=0}\,, \quad (17)$$

for all $x \in [0,1]$.

Theorem 2

Assume the basic situation holds as established in the previous sections. Suppose also that assumptions (a)-(f) hold. Then, assuming $\phi_{or''}=\max$,

$$\begin{aligned}\phi_{+\infty}(\underline{\theta}|\hat{Z}^{(i,j)}_m;R,P) &\overset{d}{=} \lim_{p\to+\infty}(\phi_p(\underline{\theta}|\hat{Z}^{(i,j)}_m;R,P))\\ &= 1-h^{-1}(\min(\rho(\underline{\theta}),h(0)))\,,\end{aligned} \quad (18)$$

where

$$\rho(\underline{\theta}) \overset{d}{=} \max_{(Z''^{(i,j)}_m \in D''^{(i,j)}_m)} (\alpha(\underline{\theta},Z''^{(i,j)}_m))\,, \quad (19)$$

$$\alpha(\underline{\theta},Z''^{(i,j)}_m) \overset{d}{=} \kappa\cdot E_V(\nu(\phi_H(\underline{\theta},V,Z''^{(i,j)}_m)))\,, \quad (20)$$

where $E_V(\cdot)$ denotes ordinary statistical expectation with respect to random vector V (replacing the argument non-random $Z'^{(i,j)}_m$) which has probability density function f where at any value $Z'^{(i,j)}_m$ for V, f has the conditional form

$$f(Z'^{(i,j)}_m|\hat{Z}'^{(i,j)}_m) \overset{d}{=} \prod_{\substack{k=1,\ldots,M'\\ \ell=i,j}}\left(f^{(\ell)}_{k,m}(Z^{(\ell)}_{k,m}|\hat{Z}^{(\ell)}_{k,m})\right). \quad (21)$$

(Proof:

Use the canonical expansion for $\phi_{or'}$ as in (15) where q is replaced by a summation over $D'^{(i,j)}_{m,p}$ and x_ω is replaced by $G_p(\underline{\theta},Z^{(i,j)}_m)$. In turn, expand $h(1-G_p(\underline{\theta},Z^{(i,j)}_m))$ in terms of variable $\phi_p(Z'^{(i,j)}_m|\hat{Z}'^{(i,j)}_m)$ around 0, obtaining

$$h(1-G_p(\underline{\theta},Z^{(i,j)}_m)) = \nu(\phi_H(\underline{\theta},Z^{(i,j)}_m))\cdot\phi_p(Z'^{(ij)}_m|\hat{Z}'^{(ij)}_m) + O((\phi_p(Z^{(ij)}_m|\hat{Z}^{(ij)}_m))^2).$$

Also expanding for $\Delta^{(\ell)}_{k,m,p}$ small,

$$\phi_p(Z'^{(ij)}_m|\hat{Z}'^{(ij)}_m) = \kappa\cdot f(Z'^{(ij)}_m|\hat{Z}'^{(ij)}_m)\cdot\Delta^{(ij)}_{m,p} + O((\Delta^{(ij)}_{m,p})^2)\,,$$

where

$$\Delta^{(ij)}_{m,p} = \prod_{\substack{k=1,\ldots,M'\\ \ell=i,j}}\left(\Delta^{(\ell)}_{k,m,p}\right).$$

The result then follows from the definition of an integral.)

Remark.

An important family of t-norms and t-conorms is due to Frank [13]. (See also [12].) These satisfy the basic modular relation

$$\phi_{or}(x,y) = x + y - \phi_\&(x,y)\,, \quad (22)$$

for all $x,y \in [0,1]$. Frank has shown that this family can be characterized by the Archimedean class - i.e., all $\phi_\&$, ϕ_{or} such that for all $x \in (0,1)$,

$$\phi_\&(x,x)<x \text{ and } \phi_{or}(x,x)>x\,, \quad (23)$$

which is also DeMorgan, i.e.

$$\phi_{or}(x,y) = 1 - \phi_{\&}(1-x,1-y)\ , \tag{24}$$

for all $x,y \in [0,1]$, and as well by the class of all ordinal sums (typesof affine transforms involving the block diagonal parts of $[0,1]^2$ -see [9],[12],[13]) of subsets of the Archimedean class. Indeed, the Archimedean class of Frank contains many common t-norms and t-conorms and can be conveniently parameterized, using parameter s :

$$\phi_{\&,s}(x_1,\ldots,x_q) \stackrel{d}{=} \log_s\Big(1+\big(\prod_{\omega=1}^{q} (s^{x_\omega}-1)\big)/(s-1)^{q-1}\Big) \tag{25}$$

with $\phi_{or}(x_1,\ldots,x_q)$ determined from the DeMorgan or modular relations, for all $x_1,\ldots,x_q \in [0,1]$, with generator function h_s given as

$$h_s(x) = -\log((s^x-1)/(s-1));\ x \in [0,1], \tag{26}$$

for all $0<s\le+\infty$, where the special cases are :

s=0 (non-Archimedean) :

$$\begin{aligned} &\phi_{\&,0}(x_1,\ldots,x_q) = \min(x_1,\ldots,x_q)\ , \\ &\phi_{or,0}(x_1,\ldots,x_q) = \max(x_1,\ldots,x_q)\ , \\ &h_0(x) = \begin{cases} +\infty\ , & \text{if } x=0 \\ 0\ , & \text{if } 0<x\le 1 \end{cases} \end{aligned} \tag{27}$$

s=1:

$$\begin{aligned} &\phi_{\&,1}(x_1,\ldots,x_q) = x_1\cdots x_q\ , \\ &\phi_{or,1}(x_1,\ldots,x_q) = \mathrm{probsum}(x_1,\ldots,x_q) = 1 - \prod_{1\le\omega\le q}(1-x_\omega)\ , \\ &h_1(x) = -\log(x)\ ;\ \text{all } x \in [0,1] \end{aligned} \tag{28}$$

$s=+\infty$:

$$\begin{aligned} &\phi_{\&,+\infty}(x_1,\ldots,x_q) = \min(x_1+\cdots+x_q,1), \\ &\phi_{or,+\infty}(x_1,\ldots,x_q) = \max(x_1+\cdots+x_q-(q-1),0) \\ &h_{+\infty}(x) = 1-x\ ;\ \text{all } x \in [0,1]. \end{aligned} \tag{29}$$

It follows that if all t-norms and t-conorms used in the computations for the posterior function of θ are culled from Frank's family and the assumptions (a),(b),(f) hold, then Theorem 2 is valid, and the key computations for κ and ν are :

For generator $h_{s'}$ of $\phi_{or',s'}$,

$$(d\, h_{s'}(x)/dx)_{x=1} = \begin{cases} -(\log s')\cdot s'/(s'-1) & (0<s'<+\infty, s'\ne1) \\ -1 & (s'=1) \end{cases}\ . \tag{30}$$

For $\phi_{\&,s}$,

$$(\partial\, \phi_{\&,s}(x,y)/\partial y)_{y=0} = \begin{cases} (s^x-1)/(s-1) & (0<s<+\infty, s\ne1) \\ x & (s=1) \end{cases}\ . \tag{31}$$

For $\phi_{\&_3,s'} = \phi_{\&',s'}$,

$$\kappa = \begin{cases} ((\log s')/(s'-1))^{2M'-1} & (0<s'<+\infty, s'\ne1) \\ 1 & (s'=1) \end{cases}\ . \tag{32}$$

Finally, let us consider briefly the effect of the choice of inference rules and the accuracy of the error distributions upon the asymptotic posterior function for θ as given in Theorem 2. So far, matching tables have not been discussed. In general, although the basic structures of the inference rules do not depend upon time and the particular pair of track histories in question, the matching table does. For subjective attributes, these could be chosen as some type of symmetrization of the corresponding error distributions, while for statistical attributes, the matching tables can be naturally chosen as

$$M_{k,m}^{(i,j)} = 1-F_{n_k}(\lambda(Z_{k,m}^{(i,j)}))\ , \tag{33}$$

where

$$\lambda(Z_{k,m}^{(ij)}) = (Z_{k,m}^{(i)}-Z_{k,m}^{(j)})^T(\Sigma_{km}^{(i)}+\Sigma_{km}^{(j)})^{-1}(Z_{k,m}^{(i)}-Z_{k,m}^{(j)}) \tag{34}$$

and F_{n_k} is the probability distribution function of λ corresponding to the $Z_{k,m}^{(\ell)}$ being r.v.'s with a common expectation, but otherwise described by independent p.d.f.'s $f_{k,m}^{(\ell)}$ as for V in Theorem 2.

Note also by inspection that $\phi_H(\underline{\theta},Z_m^{(ij)})$ is a non-decreasing function in the variables $\underline{\lambda}(Z_m^{\prime(ij)}) \stackrel{d}{=} (\lambda(Z_{1m}^{(ij)}),\ldots,\lambda(Z_{M',m}^{(ij)}))$, provided that we choose, for example, $\phi_{mod_{v,k}}(x)=x^{a_{v,k}}$ and $\phi_{mod_v}(x)=x^{b_v}$, for some constant positive $a_{v,k}$, b_v ; all $x\in[0,1]$. It then easily follows, that if all error distributions are not in dirac-form, but indicate some spread or confusion between observed/smoothed data, then if we can legitimately obtain for the inference rules the consistency structure given by all $a_{v,k}$ approaching the extreme extensification 0, with all b_v approaching the extreme intensification $+\infty$, for $0\le\theta<1$, $\phi(\theta|Z^{(ij)};R)$ approaches 0 for fixed values of $\underline{\lambda}$, and similarly for $\phi_H(\underline{\theta},Z_m^{(ij)})$. On the other hand, for $\underline{\theta}=1$, the latter is maximized. The consequence of this,applying standard inequalities to the expectation $E_V(\cdot)$ in Theorem 2 and making the additional mild assumption (true for Frank's family for $s\ge1$) that $\nu(x)$ is non-decreasing in x over $[0,1]$ with $\nu(0)=0,\nu(1)=1$,is that $\phi(\theta|\mathrm{diag}_{\cdot}Z_m^{(ij)};R,P)$ approaches the possibility function ϕ_W representing complete data association between i and j, i.e.

$$\phi_W(\theta) = \begin{cases} 1 & \text{iff } \theta=1 \\ 0 & \text{iff } 0\le\theta<1 \end{cases}\ . \tag{35}$$

Future work will be directed toward extending, further quantifying, and establishing empirical bases for the above study.

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